I. GENERAL COURSE INFORMATION

Subject and Number: Mathematics 110
Descriptive Title: Structures and Concepts in Mathematics
Course Disciplines: Mathematics
Division: Mathematical Sciences

Catalog Description:
As an introduction to the use of logical, quantitative, and spatial reasoning in the discipline of mathematics, students in this course examine the mathematical topics of set theory, numeration, number theory, functions, graphs, patterns and the structure of real numbers. Students investigate the interrelationships among these topics, with an emphasis on algebraic, geometric and kinesthetic modeling, inductive and deductive logic, and proofs using pictures. Designed for pre-service elementary school teachers, this course is appropriate for all students interested in a deeper understanding of the structure of mathematics.

Note: The maximum UC credit allowed for students completing Mathematics 110, 115, and 116 is one course.

Conditions of Enrollment:
Prerequisite: Mathematics 67 or Mathematics 73 or Mathematics 80 with a minimum grade of C in prerequisite or qualification by appropriate assessment

Course Length:	**X Full Term**	**Other (Specify number of weeks):**
Hours Lecture:	**2.00 hours per week**	**TBA**
Hours Laboratory:	**3.00 hours per week**	**TBA**
Course Units:	**3.00**	

Grading Method: Letter
Credit Status: Associate Degree Credit

Transfer CSU: X Effective Date: Prior to July 1992
Transfer UC: X Effective Date: Fall 2001

General Education:
El Camino College:
4B – Language and Rationality – Communication and Analytical Thinking
Term: Other:
6 – Mathematics Competency
Term: Other:

CSU GE:
B4 - Mathematics/Quantitative Thinking
Term: Spring 2006 Other:

IGETC:

II. OUTCOMES AND OBJECTIVES

A. COURSE STUDENT LEARNING OUTCOMES (The course student learning outcomes are listed below, along with a representative assessment method for each. Student learning outcomes are not subject to review, revision or approval by the College Curriculum Committee)

SLO #1 Perform and Interpret Basic Operations
Students will be able to demonstrate/perform the four basic operations with real numbers and interpret the results.

SLO #2 Explain Mathematical Concepts
Students will be able to explain the underlying mathematical concepts of the binary operations using written and oral means.

SLO #3 Solve Application Problems
Students will be able to solve an application problem and design an application when parameters are given.

B. Course Student Learning Objectives (The major learning objective for students enrolled in this course are listed below, along with a representative assessment method for each)

1. Demonstrate the strengths and weaknesses of various numeration systems, including the advantages of place-value systems.
   Oral exams
2. Perform binary operations on whole numbers in a variety of numeration systems.
   Written homework
3. Construct, use and analyze algebraic, geometric and kinesthetic representations for binary operations on integers (in a variety of bases) and rational numbers.
   Oral exams
4. Present picture proofs justifying the common rules of divisibility.
   Oral exams
5. Utilize set operations (union, intersection and complements) and their algebraic properties, as well as Venn diagrams, to solve logical and arithmetic problems.
   Homework Problems
6. Attack and solve application problems with systematic and creative problem solving strategies (Polya's problem solving guidelines).
   Homework Problems
7. Recognize, model, and solve pattern problems, including arithmetic and geometric patterns and sequences, using inductive or deductive reasoning.
   Performance exams
8. Model and analyze application problems using graphs of discrete, linear, step, quadratic and exponential functions.
   Performance exams
9. Utilize modular arithmetic in problem solving.
   Homework Problems
10. Decide on and execute a reasonable and meaningful strategy for estimating length, distance, perimeter and area in real-world contexts.
    Performance exams

III. OUTLINE OF SUBJECT MATTER (Topics are detailed enough to enable a qualified instructor to determine the major areas that should be covered as well as ensure consistency from instructor to instructor and semester to semester.)

Lecture or Lab	Approximate Hours	Topic Number	Major Topic
Lecture	4	I	Mathematical Reasoning A. Four-step problem solving process (Polya) B. Inductive and deductive logic, proofs by induction, proofs by picture C. Algebraic, geometric and kinesthetic models
Lab	6	II	Mathematical Reasoning Activities based on A. Four-step problem solving process (Polya) B. Inductive and deductive logic, proofs by induction, proofs by picture C. Algebraic, geometric and kinesthetic models
Lecture	2	III	Numeration Systems A. Roman, Egyptian, Babylonian and Hindu-Arabic numeration systems and place-value systems of numeration B. Base systems, conversion between bases, arithmetic operations in bases other than base ten
Lab	4	IV	Numeration Systems Activities based on A. Roman, Egyptian, Babylonian and Hindu-Arabic numeration systems and place-value systems of numeration B. Base systems, conversion between bases, arithmetic operations in bases other than base ten
Lecture	6	V	Set Theory A. Definitions of sets, subsets, elements, equal sets, the null set and universal sets, and set notation B. Injective and surjective mappings between sets, one-to-one correspondences C. Operations on sets (union, intersection and complement) and the properties of these operations (commutative, associative and distributive properties) D. Structure of Venn Diagrams
Lab	9	VI	Set Theory Activities based on A. Definitions of sets, subsets, elements, equal sets, the null set and universal sets, and set notation B. Injective and surjective mappings between sets, one-to-one correspondences C. Operations on sets (union, intersection and complement) and the properties of these operations (commutative, associative and distributive properties) D. Structure of Venn Diagrams
Lecture	5	VII	Number Theory A. Divisibility: definitions and theorems B. Divisibility rules in base 10 and other bases C. Prime, composite numbers and prime factorization

			D. Greatest common divisors and least common multiples E. Modular arithmetic
Lab	8	VIII	Number Theory Activities based on A. Divisibility: definitions and theorems B. Divisibility rules in base 10 and other bases C. Prime, composite numbers and prime factorization D. Greatest common divisors and least common multiples E. Modular arithmetic
Lecture	3	IX	Patterns A. Arithmetic and geometric patterns and sequences B. Recursively defined sequences, such as the Fibonacci sequence
Lab	5	X	Patterns Activities based on A. Arithmetic and geometric patterns and sequences B. Recursively defined sequences, such as the Fibonacci sequence
Lecture	6	XI	Functions and Graphs A. Verbal, algebraic, tabular and graphical representations of functions B. Coordinate geometry C. Graphs of discrete, linear, step, quadratic and exponential functions
Lab	8	XII	Functions and Graphs Activities based on A. Verbal, algebraic, tabular and graphical representations of functions B. Coordinate geometry C. Graphs of discrete, linear, step, quadratic and exponential functions
Lecture	4	XIII	Estimation and Measurement A. Estimations of arithmetic operations on whole numbers, fractions and decimals B. Techniques for estimating measurements, such as length, distance, perimeter and area
Lab	6	XIV	Estimation and Measurement Activities based on A. Estimations of arithmetic operations on whole numbers, fractions and decimals B. Techniques for estimating measurements, such as length, distance, perimeter and area
Lecture	4	XV	Real Number System A. Whole numbers and counting numbers: definition, ordering, operations and properties B. Integers: definition, ordering, operations and properties C. Rational numbers: definition, relative size, ordering, operations and properties D. Irrational numbers: definition E. Properties of real numbers: closure, commutative, associative, identity and inverse, and the distributive

			properties (multiplication over addition and subtraction, and exponentiation over multiplication and division) F. Models for teaching real numbers
Lab	8	XVI	Real Number System Activities based on A. Whole numbers and counting numbers: definition, ordering, operations and properties B. Integers: definition, ordering, operations and properties C. Rational numbers: definition, relative size, ordering, operations and properties D. Irrational numbers: definition E. Properties of real numbers: closure, commutative, associative, identity and inverse, and the distributive properties (multiplication over addition and subtraction, and exponentiation over multiplication and division) F. Models for teaching real numbers
Lecture	2	XVII	Mathematical Education Resources A. California Standards, National Council of Teachers of Mathematics Standards, and related standards and organizations B. Resources for mathematics and mathematics teaching
Total Lecture Hours		36	
Total Laboratory Hours		54	
Total Hours		90	

IV. PRIMARY METHOD OF EVALUATION AND SAMPLE ASSIGNMENTS

A. PRIMARY METHOD OF EVALUATION:

Problem solving demonstrations (computational or non-computational)

B. TYPICAL ASSIGNMENT USING PRIMARY METHOD OF EVALUATION:

Politicians want to know how their position on an issue is viewed by particular constituencies. Decisions about policies are often made on the basis of polling information. Suppose an opinion survey was conducted to determine how much support there was for the president's policies. People were asked three questions: (1) Do you support the president's economic policy? (2) Do you support the president's foreign policy? (3) Do you support the president's social policy? (a) Sketch a Venn Diagram where set E contains the people responding "Yes" to question (1); set F contains the people responding "Yes" to question (2); and set S contains the people responding "Yes" to question (3). There are 29 people in set E, 49 in set F, and 24 in set S. Twelve people responded "Yes" to all three questions. Twenty people answered "Yes" to questions (1) and (2); fifteen people answered "Yes" to questions (1) and (3); and sixteen answered "Yes" to questions (2) and (3). (b) What percent of the people polled agree with the president's economic policy? (c) Describe the subset that would be represented by the union of E and S. Write your answer in complete sentences. (d) If the president could make one single region of the Venn Diagram larger, which region would it be? Justify your answer.

C. COLLEGE-LEVEL CRITICAL THINKING ASSIGNMENTS:

1. Definition: A prime number is a superprime if, as digits are deleted from the right, each resulting number is itself a prime number. For example, the prime number 7331 is a superprime because 733, 73 and 7 are prime numbers. (a) For a prime number to be a superprime, what digits cannot appear in the number? Write a sentence or two justifying your answer. (b) Of the digits that can appear in a superprime, what digits cannot be the left-most digit? Write a sentence or two justifying your answer. (c) Determine all the two-digit superprimes. (d) Find at least one three-digit superprime. Explain your strategy for finding your example in a sentence or two.

2. Design and present to the class a geometric interpretation of a division of fractions problem. You may begin with a simple example. For instance, explain why $1/2 \div 1/4 = 2$, using a geometric representation. But you must move beyond examples to a general division of fractions problem. Your geometric model of division of fractions must work when using improper fractions.

D. OTHER TYPICAL ASSESSMENT AND EVALUATION METHODS:

Objective Exams
Quizzes
Written homework
Homework Problems
Other (specify):
Reflective Essays, Poster Presentations and Teaching Lessons

V. INSTRUCTIONAL METHODS

Demonstration
Discussion
Lecture
Multimedia presentations
Other (please specify)
Individual or Small Group Presentations

Note: In compliance with Board Policies 1600 and 3410, Title 5 California Code of Regulations, the Rehabilitation Act of 1973, and Sections 504 and 508 of the Americans with Disabilities Act, instruction delivery shall provide access, full inclusion, and effective communication for students with disabilities.

VI. WORK OUTSIDE OF CLASS

Study
Answer questions
Skill practice
Required reading
Problem solving activities
Written work
Journal

Estimated Independent Study Hours per Week: 5

VII. TEXTS AND MATERIALS

A. **UP-TO-DATE REPRESENTATIVE TEXTBOOKS**
Bassarear, Tom & Moss, Meg. Mathematics for Elementary School Teachers. 7th ed. Cengage Learning, 2020.

B. **ALTERNATIVE TEXTBOOKS**

C. **REQUIRED SUPPLEMENTARY READINGS**

D. **OTHER REQUIRED MATERIALS**

VIII. CONDITIONS OF ENROLLMENT

A. Requisites (Course and Non-Course Prerequisites and Corequisites)

Requisites	Category and Justification
Course Prerequisite Mathematics-67 or	Sequential
Course Prerequisite Mathematics-73 or	Sequential
Course Prerequisite Mathematics-80 or	Sequential
Non-Course Prerequisite	Placement assessment is an officially recognized mechanism for controlling enrollment in developmental mathematics courses. Placement cut scores are periodically reviewed by faculty and adjusted to match success rates in the target courses. Students who do not meet the placement cut score for this class are statistically highly unlikely to succeed.

B. Requisite Skills

Requisite Skills
Solve linear, quadratic, and rational equations. (Mathematics 73 and Mathematics 80) MATH 73 - Solve problems involving a variety of function types, including linear, quadratic, polynomial, rational and radical functions, as well as the absolute value function. MATH 80 - Solve problems involving a variety of function types, including linear, quadratic, polynomial, rational, radical, exponential, and logarithmic functions. MATH 67 - Construct and use equations and inequalities to represent relationships involving one or more unknown or variable quantities to solve problems.
Graph linear, quadratic, and rational functions. (Mathematics 73 and Mathematics 80) MATH 80 - Graph a variety of functions and relations and draw connections between these graphs and solutions to problems. MATH 73 - Graph a variety of functions and relations and draw connections between these graphs and solutions to problems. MATH 67 - Translate problems from a variety of contexts into a mathematical representation (symbolic, tabular, and graphic) and vice versa. MATH 67 - Describe the behavior of linear and exponential functions using symbolic expressions, verbal descriptions, tables and graphs.

Understand and utilize variables and function notation to solve problems. (Mathematics 73 and Mathematics 80) MATH 73 - Recognize functional relationships in the form of graphs, data or symbolic equations. MATH 73 - Solve problems involving a variety of function types, including linear, quadratic, polynomial, rational and radical functions, as well as the absolute value function.
Demonstrate ability to analyze problems and solutions critically. Explain, in writing, the reasoning behind solutions of application problems. (Mathematics 73 and Mathematics 80) ENGL 1A - Read and apply critical-thinking skills to numerous published articles and to college-level, book-length works for the purpose of writing and discussion. MATH 67 - Translate problems from a variety of contexts into a mathematical representation (symbolic, tabular, and graphic) and vice versa. MATH 67 - Describe the behavior of linear and exponential functions using symbolic expressions, verbal descriptions, tables and graphs.

C. Recommended Preparations (Course and Non-Course)

Recommended Preparation	Category and Justification

D. Recommended Skills

Recommended Skills

E. Enrollment Limitations

Enrollment Limitations and Category	Enrollment Limitations Impact

Course created by Paul Wozniak on 10/01/1988.

BOARD APPROVAL DATE:

LAST BOARD APPROVAL DATE: 05/18/2020

Last Reviewed and/or Revised by: Susanne Bucher **Date:** 03/01/2020

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